Oxford PAT 2016

14. If you want to do $\sqrt{10}$ to 1 d.p. without calc, enough to say $3.1^2 = (3+0.1)^2 = 9 + 0.6 + 0.01 = 9.61$

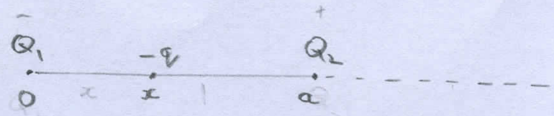
$3.2^2 = 10.24$

10.24 is closest to 10 $\therefore \sqrt{10} = 3.2$ to 1 d.p.

16. Generally,


$\underline{Q_1, Q_2 > 0}$ $\quad \underline{F} = -k \dfrac{qQ}{r^2} \hat{\underline{r}}$ where $\hat{\underline{r}}$ points from $Q$ to $q$

$\therefore$




$\therefore \quad \underline{F} = kq\left[-\dfrac{Q_1}{x^2}\hat{\underline{r}}_1 - \dfrac{Q_2}{(a-x)^2}\hat{\underline{r}}_2\right]$ $\quad \hat{\underline{r}}_1$ points from $Q_1$ to $q$; $\hat{\underline{r}}_2$ " " $Q_2$ " $q$

Now, if $0 < x < a$, $\hat{\underline{r}}_1 = -\hat{\underline{r}}_2$ ①

whereas if $x < 0$ or $x > a$, $\hat{\underline{r}}_1 = \hat{\underline{r}}_2$ ②

So for case ①, $\underline{F} = 0 \Rightarrow \dfrac{Q_1}{x^2} = \dfrac{Q_2}{(a-x)^2}$

$\therefore \left(\dfrac{x-a}{x}\right)^2 = \dfrac{Q_2}{Q_1} \Rightarrow \dfrac{x-a}{x} = \pm\sqrt{\dfrac{Q_2}{Q_1}} = 1 - \dfrac{a}{x}$

$\therefore \dfrac{a}{x} = 1 \pm \sqrt{\dfrac{Q_2}{Q_1}} \quad \therefore x = \dfrac{a}{1 \pm \sqrt{\dfrac{Q_2}{Q_1}}}$ ③

clearly + sol.$^n$ gives $x > 0$ and $x < a$ as required

− sol.$^n$: $x = \dfrac{a\sqrt{Q_1}}{\sqrt{Q_1} - \sqrt{Q_2}}$ : if $Q_2 < Q_1$, $x > 0$ & $x > a$; if $Q_2 > Q_1$, $x < 0$ } contradicts assumption $0 < x < a$ $\therefore$ not a sol.$^n$

Case ②, $\dfrac{Q_1}{x^2} + \dfrac{Q_2}{(a-x)^2} = 0 \rightarrow$ no sol.$^n$

$\therefore \quad x = \dfrac{a}{1 + \sqrt{\dfrac{Q_2}{Q_1}}}$

If $Q_1/Q_2 < 0$ $\quad$ clearly no sol.$^n$ $0 < x < a$ $(\hat{\underline{r}}_1 = \hat{\underline{r}}_2)$

otherwise $\hat{\underline{r}}_1 = -\hat{\underline{r}}_2$ and ③ is again obtained : + sol$^n$ gives $0 < x < a$ ∴ contradicts otherwise

↳ − sol.$^n$ gives either $x < 0$ or $x > a$ as required.

$\therefore$ − sol.$^n$ is if $\dfrac{Q_1}{Q_2} < 0$